LAST NAME: ______________________

FIRST NAME: ______________________

**Problem 1** (a) Calculate the image of the sequence $\langle 3, 0, 2\rangle$ under Gödel numbering and show your work. If this image does not exist, prove it.

**Answer:**

$2^{3+1} \cdot 3^{0+1} \cdot 5^{2+1} =$

$2^4 \cdot 3 \cdot 5^3 = 10^3 \cdot 6 =$

$\boxed{6000}$

(b) Calculate the pre-image of the number 2940 under Gödel numbering and show your work. If this pre-image does not exist, prove it.

**Answer:**

$2940 = 10 \cdot 294 =$

$10 \cdot 3 \cdot 98 =$

$2 \cdot 3 \cdot 5 \cdot 2 \cdot 49 =$

$2^2 \cdot 3^1 \cdot 5^1 \cdot 7^2$

answer:

$\boxed{\langle 1, 0, 0, 1 \rangle}$

(c) Calculate the pre-image of the number 3850 under Gödel numbering and show your work. If this pre-image does not exist, prove it.

**Answer:**

$3850 = 385 \cdot 10 =$

$2 \cdot 5 \cdot 7 \cdot 55 =$

$2 \cdot 5^2 \cdot 7 \cdot 11$

not a Gödel number, misses 3, no pre-image.

(d) Let $m$ be the image of the sequence

$$s = \langle x_1, x_2, x_3, x_4, x_5, x_6\rangle$$

(with 2, 3, 5, 7, 11, 13 written above $x_1, \dots, x_6$)

under Gödel numbering. Represent the pre-image of the number $78\,m$ as a function of the (components of) sequence $s$. If such a representation does not exist, prove it.

**Answer:**

$78\, m = 6 \cdot 13 =$

$= 2 \cdot 3 \cdot 13$

answer:

$\boxed{\langle x_1+1, x_2+1, x_3, x_4, x_5, x_6+1\rangle}$

(e) Let $n$ be the image of the sequence

$$\langle x_1, x_2, x_3, x_4\rangle.$$

under Gödel numbering. Represent the image of the sequence

$$\langle x_1{+}2,\ x_2{+}1,\ x_3,\ x_4,\ 1\rangle$$

(with 2, 3, 5, 7, 11 written above the components)

as a function of $n$. If such a representation does not exist, prove it.

**Answer:**

answer

$n \cdot 2^2 \cdot 3^1 \cdot 11^2 =$

$= 12 \cdot 121\, n$

$= \boxed{1452\, n}$

LAST NAME: ______________________________

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**Problem 1** (a) Calculate the image of the sequence $\langle 5, 0, 1\rangle$ under Gödel numbering and show your work. If this image does not exist, prove it.

**Answer:**

$2^{5+1} \cdot 3^{0+1} \cdot 5^{1+1} =$

$2^6 \cdot 3 \cdot 5^2 = 10^2 \cdot 3 \cdot 16$

$= \boxed{4800}$

(b) Calculate the pre-image of the number 2730 under Gödel numbering and show your work. If this pre-image does not exist, prove it.

**Answer:**

$2730 = 10 \cdot 273 =$

$2 \cdot 5 \cdot 3 \cdot 91 =$

$2 \cdot 3 \cdot 5 \cdot 7 \cdot 13$

misses 11, not a Gödel number, no pre-image

(c) Calculate the pre-image of the number 6930 under Gödel numbering and show your work. If this pre-image does not exist, prove it.

**Answer:**

$6930 = 10 \cdot 693 =$

$= 10 \cdot 9 \cdot 77 =$

$= 2 \cdot 5 \cdot 3^2 \cdot 7 \cdot 11 =$

$= 2^1 \cdot 3^2 \cdot 5^1 \cdot 7^1 \cdot 11^1$

answer: $\boxed{\langle 0, 1, 0, 0, 0 \rangle}$

(d) Let $n$ be the image of the sequence

$$\langle x_1, x_2, x_3, x_4\rangle$$

under Gödel numbering. Represent the image of the sequence

$$\langle \overset{2}{x_1+1},\ \overset{3}{x_2},\ \overset{5}{x_3+1},\ \overset{7}{x_4},\ \overset{11}{2}\rangle$$

as a function of $n$. If such a representation does not exist, prove it.

**Answer:**

$n \cdot 2 \cdot 5 \cdot 11^3$

$= 10n \cdot 11 \cdot 121$

$= 10n \cdot 1331$

$= \boxed{13310n}$

(e) Let $m$ be the image of the sequence

$$s = \langle \overset{2}{x_1}, \overset{3}{x_2}, \overset{5}{x_3}, \overset{7}{x_4}, \overset{11}{x_5}, \overset{13}{x_6}\rangle$$

under Gödel numbering. Represent the pre-image of the number $143\,m$ as a function of the (components of) sequence $s$. If such a representation does not exist, prove it.

**Answer:**

$143 = 13 \cdot 11$

answer:

$\langle x_1, x_2, x_3, x_4, x_5+1, x_6+1\rangle$

LAST NAME: ______________________

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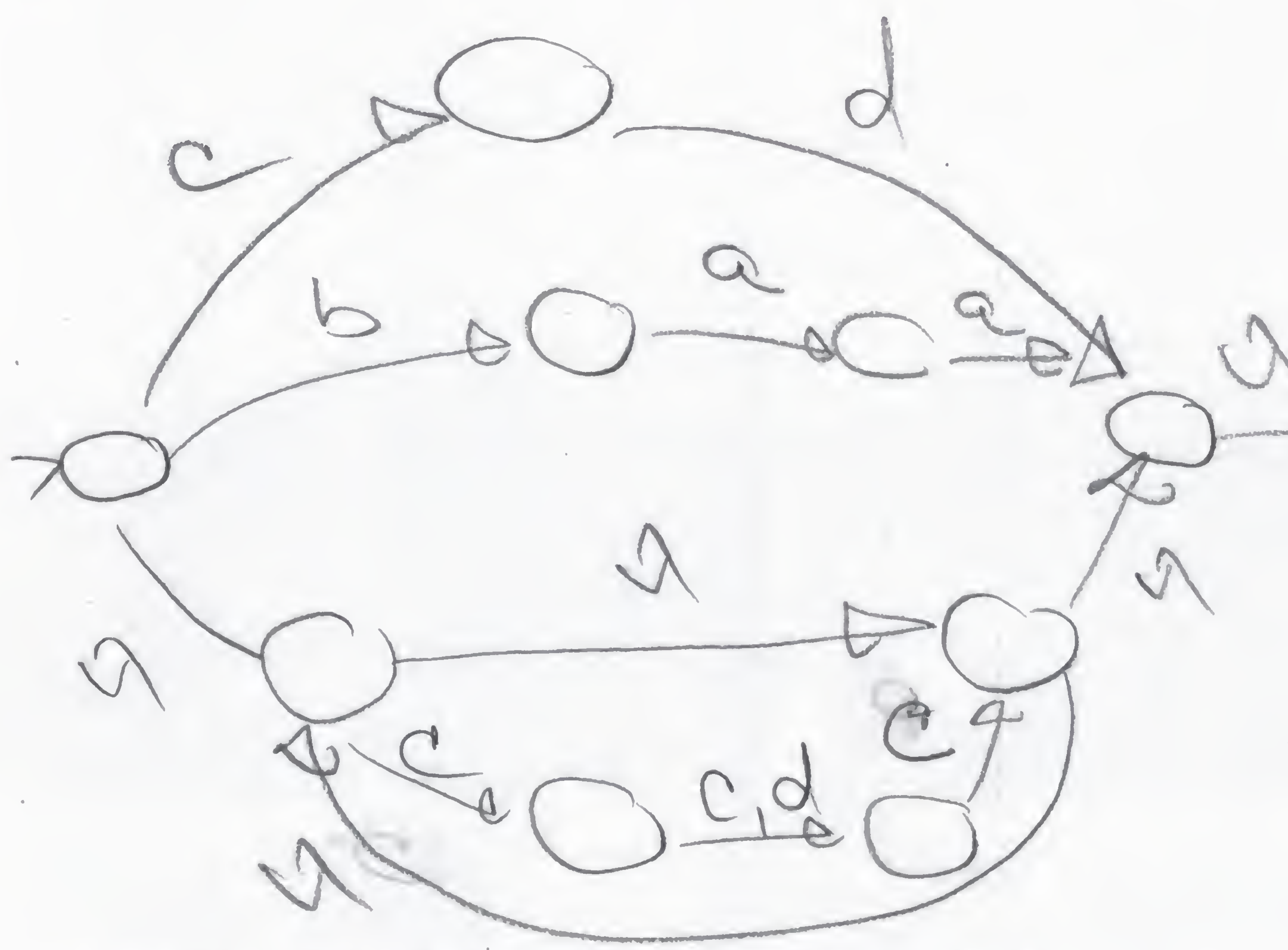
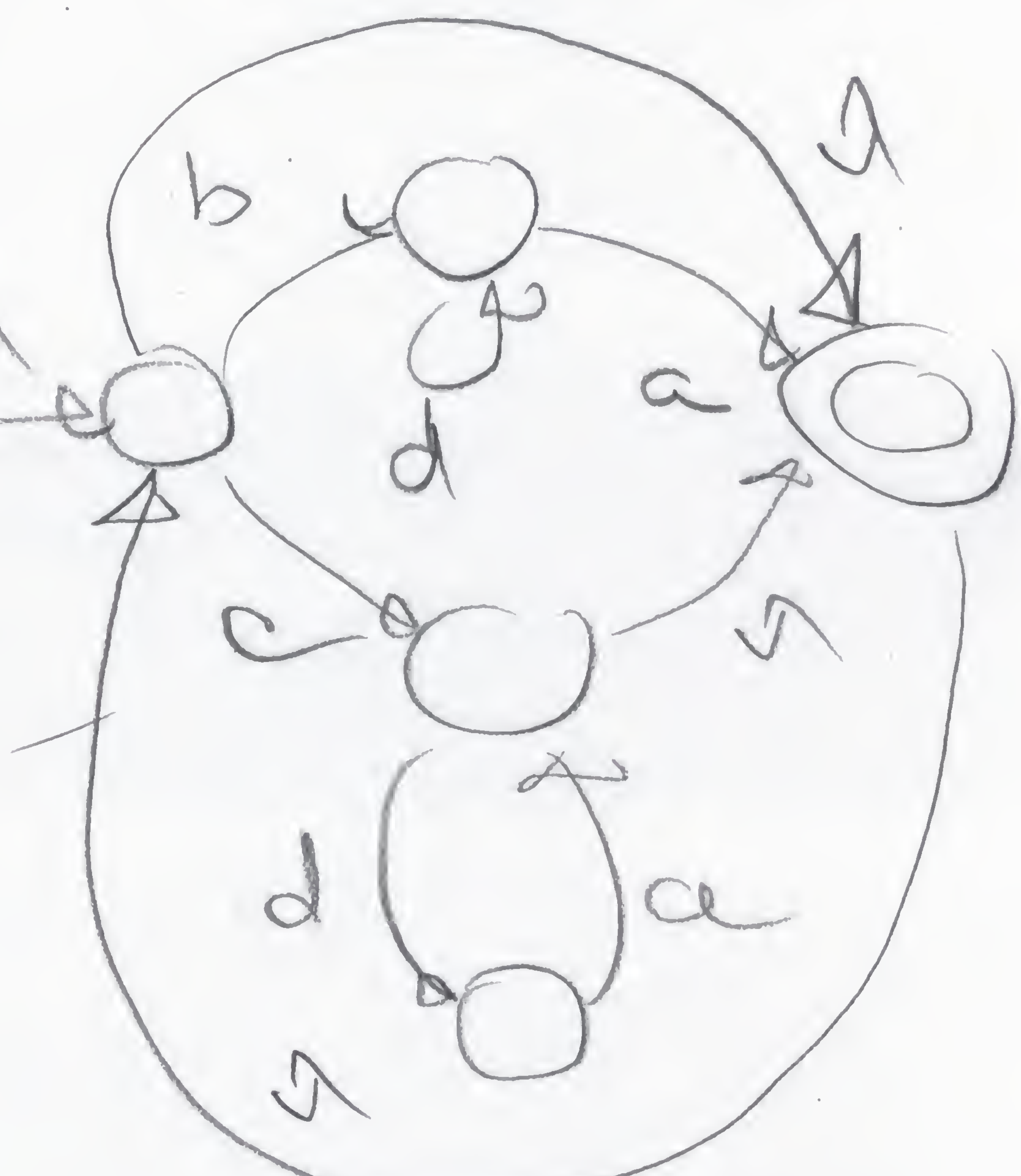
**Problem 2** Let $L$ be the language defined by the regular expression:

$$(cd \cup baa \cup (c\,(c \cup d)\,c)^*)\,(c\,(da)^* \cup bd^*a)^*$$

**(a)** Draw a state-transition graph of a finite automaton that accepts the language $L$. If such an automaton does not exist, state it and explain why.

**Answer:**





**(b)** Write a complete formal definition of a context-free grammar that generates the language $L$. If such a grammar does not exist, state it and explain why.

**Answer:**

$G = (V, \Sigma, P, S)$

$V = \{S, A, D, B, E, F\}$, $\Sigma = \{a, b, c, d\}$

$P$: $S \to AB$

$A \to cd \mid baa \mid D$

$D \to \lambda \mid DD \mid ccc \mid cdc$

$B \to \lambda \mid BB \mid cE \mid bFa$

$E \to daE \mid \lambda$

$F \to bF \mid \lambda$

**(c)** State the cardinality of the set $L$. (If $L$ is a finite set, state the exact number of elements of $L$. Otherwise, state that $L$ is infinite and specify whether it is countable or not.)

**Answer:**

L is infinite and countable.

LAST NAME: ____________________

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**Problem 2** Let $L$ be the language defined by the regular expression:

$$(\boldsymbol{a}\,(\boldsymbol{b} \cup \boldsymbol{c})^* \cup \boldsymbol{db^*c})^*\,(\boldsymbol{ab} \cup \boldsymbol{dcc} \cup (\boldsymbol{abaa})^*)$$

**(a)** Write a complete formal definition of a context-free grammar that generates the language $L$. If such a grammar does not exist, state it and explain why.

**Answer:**

$G = (V, \Sigma, P, S)$

$V = \{S, A, B, D, E, H\}$

$\Sigma = \{a, b, c, d\}$

$P$: $S \to AB$

$A \to \lambda \mid AA \mid aD \mid dEc$

$D \to \lambda \mid DD \mid b \mid c$

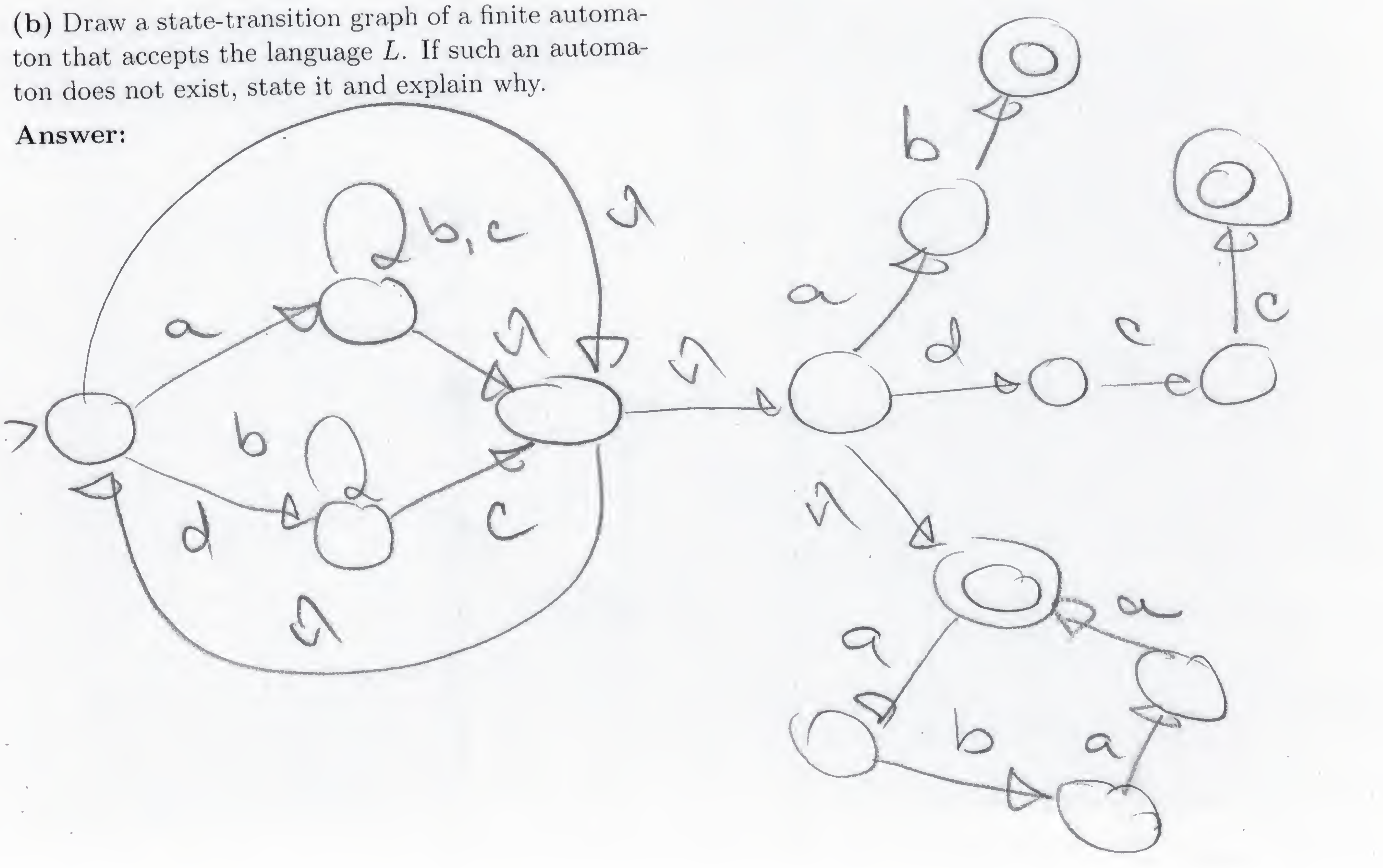
$E \to bE \mid \lambda$

$B \to ab \mid dcc \mid H$

$H \to \lambda \mid HH \mid abac$

**(b)** Draw a state-transition graph of a finite automaton that accepts the language $L$. If such an automaton does not exist, state it and explain why.

**Answer:**




**(c)** State the cardinality of the set $L$. (If $L$ is a finite set, state the exact number of elements of $L$. Otherwise, state that $L$ is infinite and specify whether it is countable or not.)

**Answer:**

L is infinite and countable

LAST NAME: ________________

FIRST NAME: ________________

**Problem 3** Let $L_1$ be the set of exactly those strings over the alphabet $\{a, b, c\}$ whose length is not greater than 3.

Let $L_2$ be the set of exactly those strings over the alphabet $\{a, b, c\}$ where the number of $a$'s is not less than 2.

**(a)** Write a regular expression that represents the language $L_1$. If such a regular expression does not exist, state it and explain why.

**Answer:** $(a \cup b \cup c \cup \Lambda)(a \cup b \cup c \cup \Lambda)(a \cup b \cup c \cup \Lambda)$

**(b)** Write a regular expression that represents the language $L_2$. If such a regular expression does not exist, state it and explain why.

**Answer:** $(b \cup c)^* a (b \cup c)^* a (a \cup b \cup c)^*$

**(c)** Write a regular expression that represents the language $L_1 \cup L_2$. If such a regular expression does not exist, state it and explain why.

**Answer:** $(a \cup b \cup c \cup \Lambda)(a \cup b \cup c \cup \Lambda)(a \cup b \cup c \cup \Lambda) \cup$
$(b \cup c)^* a (b \cup c)^* a (a \cup b \cup c)^*$

**(d)** Write a regular expression that represents the language $L_1 L_1$. If such a regular expression does not exist, state it and explain why.

**Answer:** $(a \cup b \cup c \cup \Lambda)(a \cup b \cup c \cup \Lambda)(a \cup b \cup c \cup \Lambda)(a \cup b \cup c \cup \Lambda)\cdot$
$(a \cup b \cup c \cup \Lambda)(a \cup b \cup c \cup \Lambda)$

**(e)** State the cardinality of the set $L_1$. (If $L_1$ is finite set, state the exact number of elements of $L_1$. Otherwise, state that $L_1$ is infinite and specify whether it is countable or not.)

**Answer:** $1 + 3 + 9 + 27 = \boxed{40}$

**(f)** State the cardinality of the set $L_2$. (If $L_2$ is a finite set, state the exact number of elements of $L_2$. Otherwise, state that $L_2$ is infinite and specify whether it is countable or not.)

**Answer:** infinite and counteble

**(g)** State the cardinality of the set $L_1^*$. (If $L_1^*$ is a finite set, state the exact number of elements of $L_1^*$. Otherwise, state that $L_1^*$ is infinite and specify whether it is countable or not.)

**Answer:** infinite and counteble

**(h)** State the cardinality of the set $\mathcal{P}(L_2)$ (set of subsets of $L_2$). (If $\mathcal{P}(L_2)$ is a finite set, state the exact number of its elements. Otherwise, state that $\mathcal{P}(L_2)$ is infinite and specify whether it is countable or not.)

**Answer:** infinite and uncounteble

LAST NAME: ____________________

FIRST NAME: ____________________

**Problem 3** Let $L_1$ be the set of exactly those strings over the alphabet $\{a, b, c\}$ whose length is equal to 3 or 4.

Let $L_2$ be the set of exactly those strings over the alphabet $\{a, b, c\}$ where the number of $c$'s is not less than 3.

**(a)** Write a regular expression that represents the language $L_1$. If such a regular expression does not exist, state it and explain why.

**Answer:**
$(a \cup b \cup c)(a \cup b \cup c)(a \cup b \cup c)(a \cup b \cup c \cup \lambda)$

**(b)** Write a regular expression that represents the language $L_2$. If such a regular expression does not exist, state it and explain why.

**Answer:** $(a \cup b)^* c (a \cup b)^* c (a \cup b)^* c (a \cup b \cup c)^*$

**(c)** Write a regular expression that represents the language $L_1L_1$. If such a regular expression does not exist, state it and explain why.

**Answer:**
$(a \cup b \cup c)(a \cup b \cup c)(a \cup b \cup c)(a \cup b \cup c \cup \lambda)(a \cup b \cup c)(a \cup b \cup c) \cdot$
$(a \cup b \cup c)(a \cup b \cup c \cup \lambda)$

**(d)** Write a regular expression that represents the language $L_1 \cup L_2$. If such a regular expression does not exist, state it and explain why.

**Answer:**
$(a \cup b \cup c)(a \cup b \cup c)(a \cup b \cup c)(a \cup b \cup c \cup \lambda) \cup$
$(a \cup b)^* c (a \cup b)^* c (a \cup b)^* c (a \cup b \cup c)^*$

**(e)** State the cardinality of the set $L_1$. (If $L_1$ is finite set, state the exact number of elements of $L_1$. Otherwise, state that $L_1$ is infinite and specify whether it is countable or not.)

**Answer:** $3^3 + 3^4 = 27 + 81 = \boxed{108}$

**(f)** State the cardinality of the set $L_2$. (If $L_2$ is a finite set, state the exact number of elements of $L_2$. Otherwise, state that $L_2$ is infinite and specify whether it is countable or not.)

**Answer:** infinite and countable

**(g)** State the cardinality of the set $\mathcal{P}(L_2)$ (set of subsets of $L_2$). (If $\mathcal{P}(L_2)$ is a finite set, state the exact number of its elements. Otherwise, state that $\mathcal{P}(L_2)$ is infinite and specify whether it is countable or not.)

**Answer:** infinite and uncountable

**(h)** State the cardinality of the set $L_1^*$. (If $L_1^*$ is a finite set, state the exact number of elements of $L_1^*$. Otherwise, state that $L_1^*$ is infinite and specify whether it is countable or not.)

**Answer:** infinite and countable

LAST NAME: ____________________

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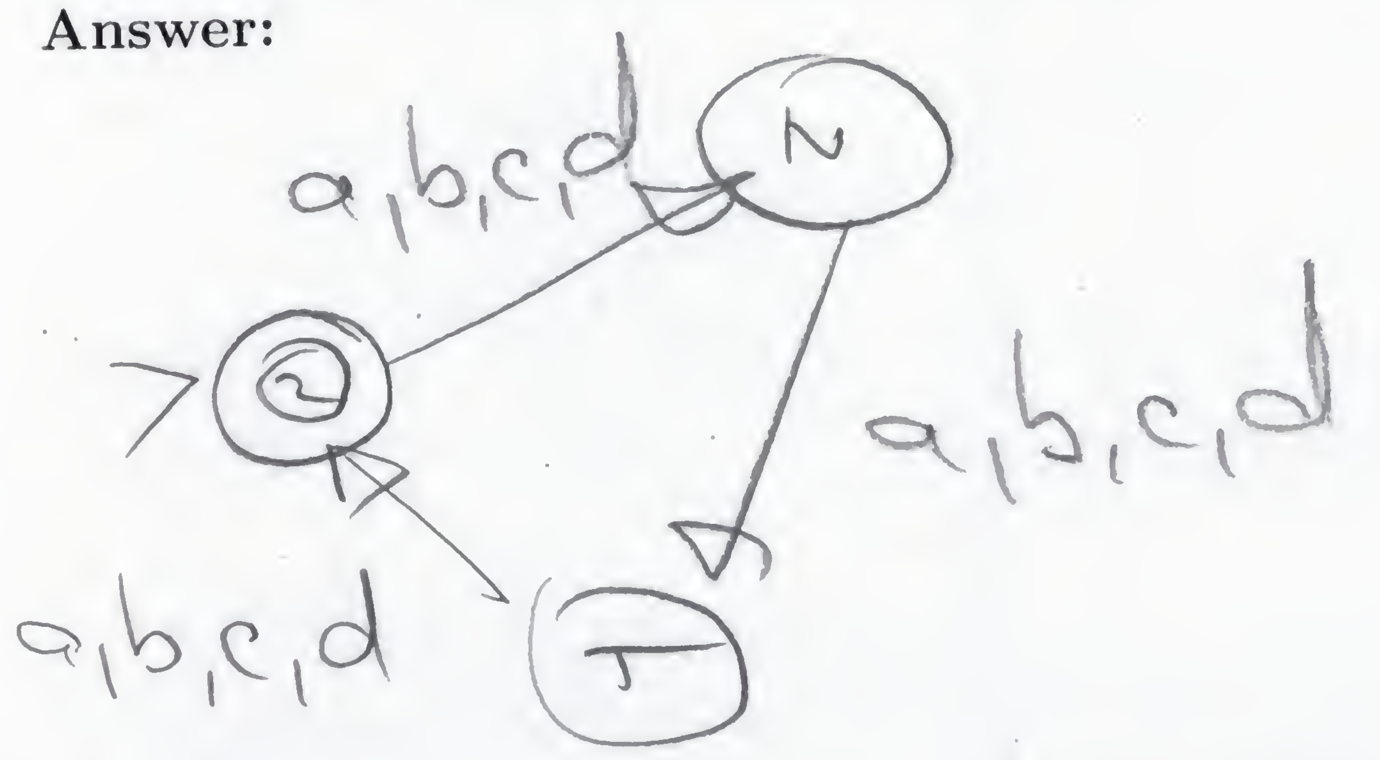
**Problem 4** Let $L_1$ be the set of exactly those strings over the alphabet $\{a, b, c, d\}$ whose length is divisible by 3.

Let $L_2$ be the set of exactly those strings over the alphabet $\{a, b, c, d\}$ where the total number of $c$'s and $d$'s (together) is odd.

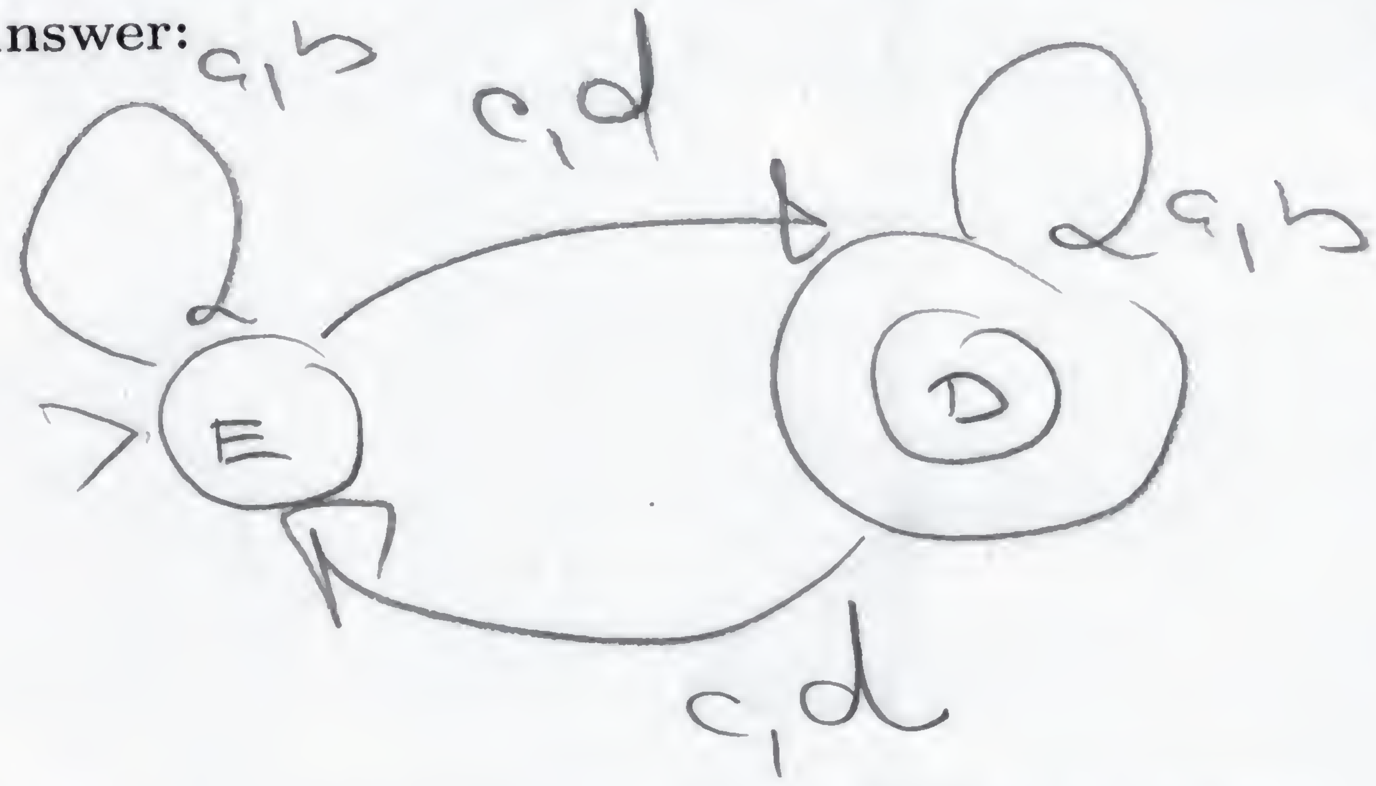
**(a)** Draw a state-transition graph of a finite automaton that accepts the language $L_1$. If such an automaton does not exist, state it and explain why.

**Answer:**



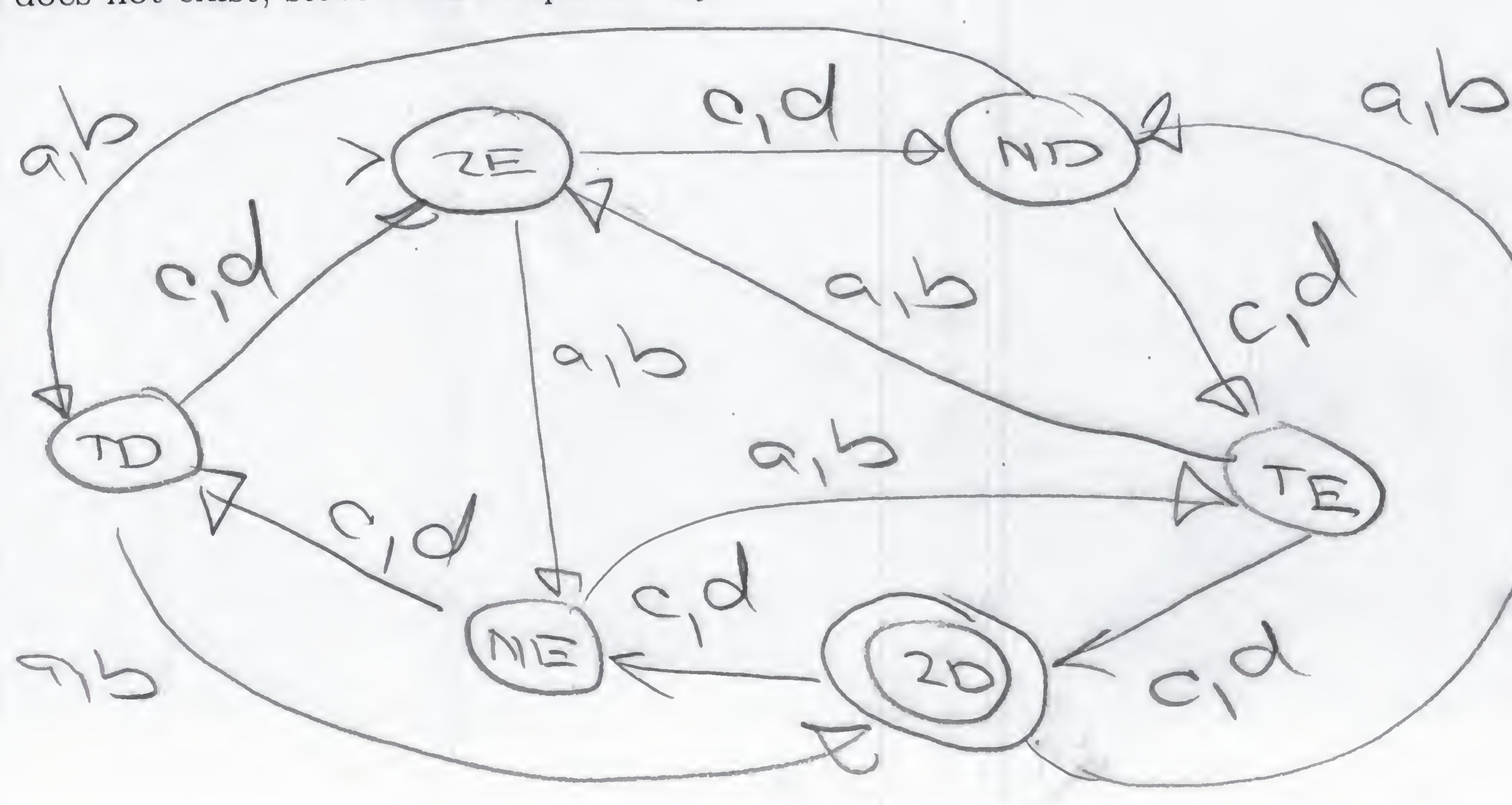

**(b)** Draw a state-transition graph of a finite automaton that accepts the language $L_2$. If such an automaton does not exist, state it and explain why.

**Answer:**



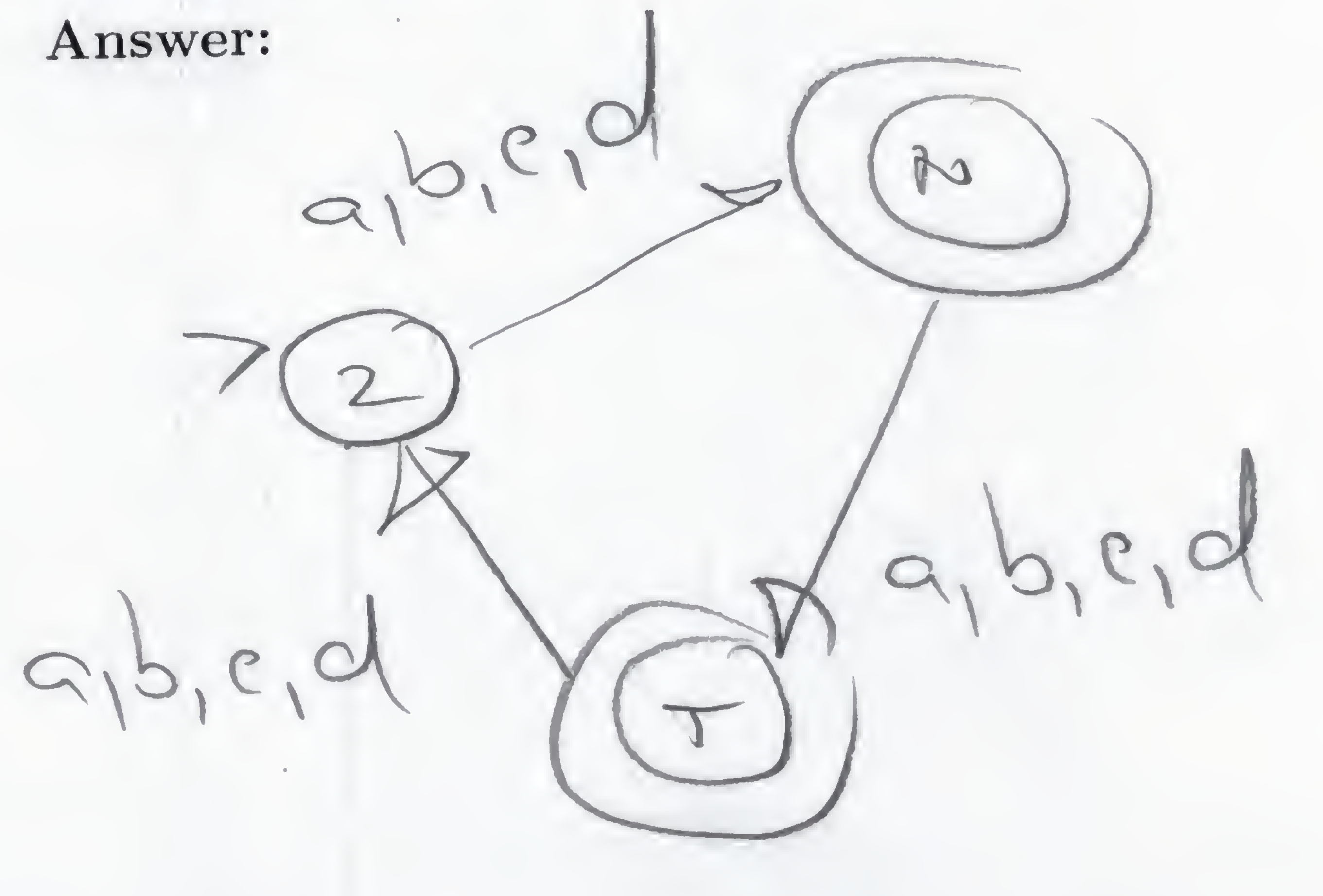

**(c)** Draw a state-transition graph of a finite automaton that accepts the language $L_1 \cap L_2$. If such an automaton does not exist, state it and explain why.

**Answer:**



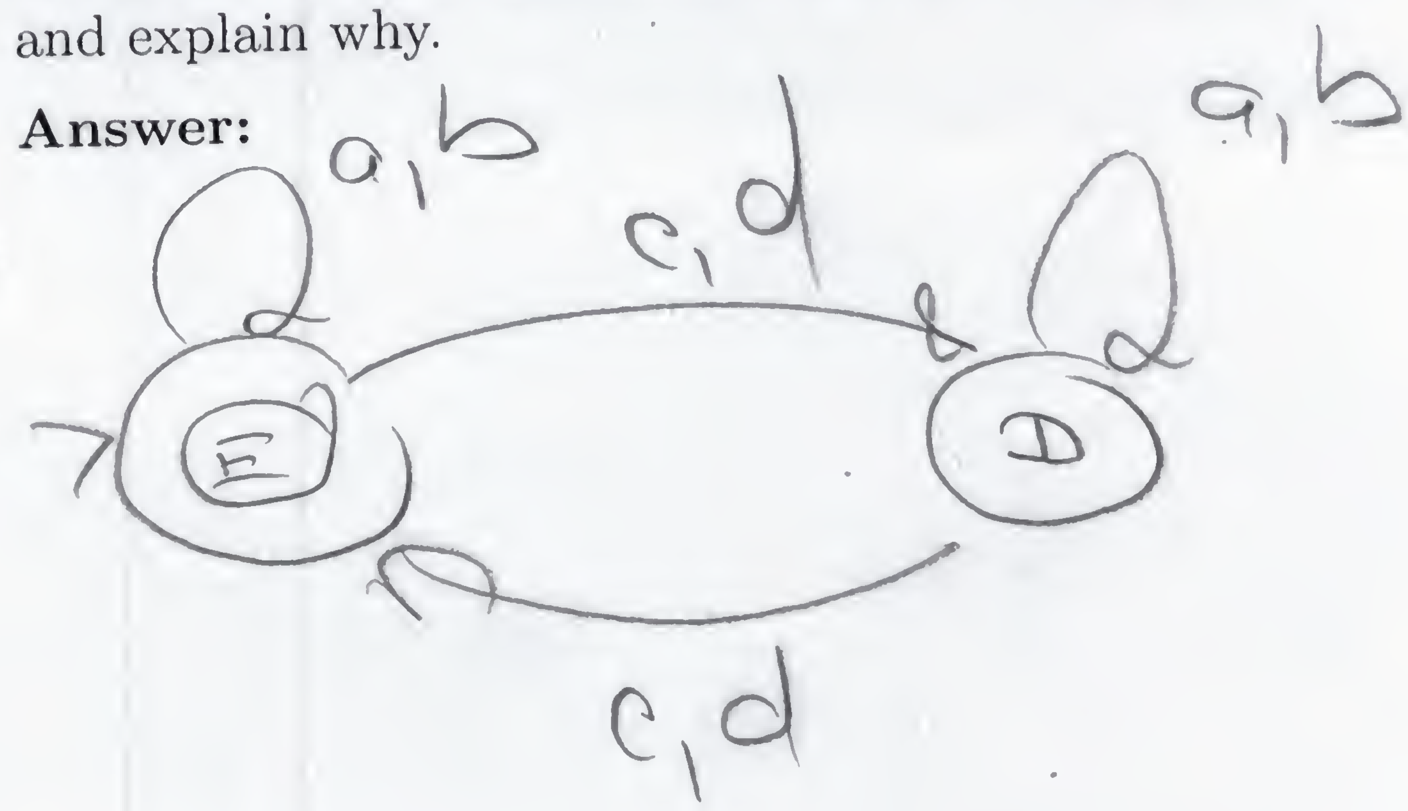

**(d)** Draw a state-transition graph of a finite automaton that accepts the language $\overline{L_1}$ (the complement of $L_1$.) If such an automaton does not exist, state it and explain why.

**Answer:**




**(e)** Draw a state-transition graph of a finite automaton that accepts the language $\overline{L_2}$ (the complement of $L_2$.) If such an automaton does not exist, state it and explain why.

**Answer:**

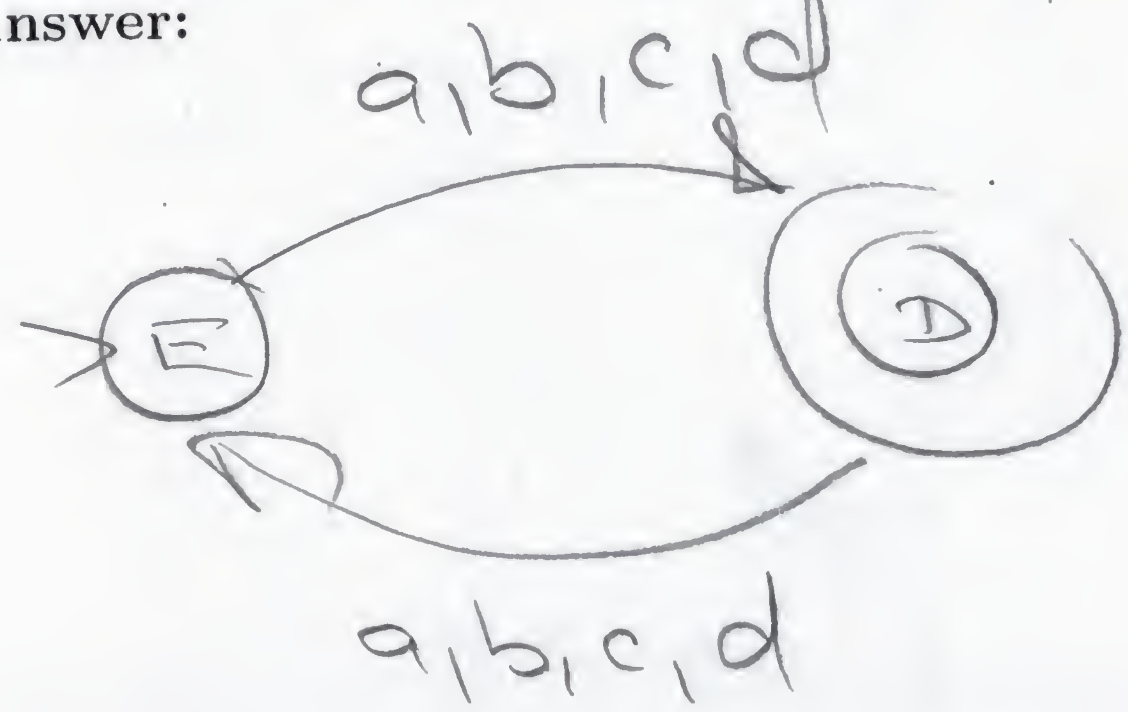


**Problem 4** Let $L_1$ be the set of exactly those strings over the alphabet $\{a, b, c, d\}$ whose length is odd.

Let $L_2$ be the set of exactly those strings over the alphabet $\{a, b, c, d\}$ where the total number of $a$'s and $c$'s (together) is divisible by 3.

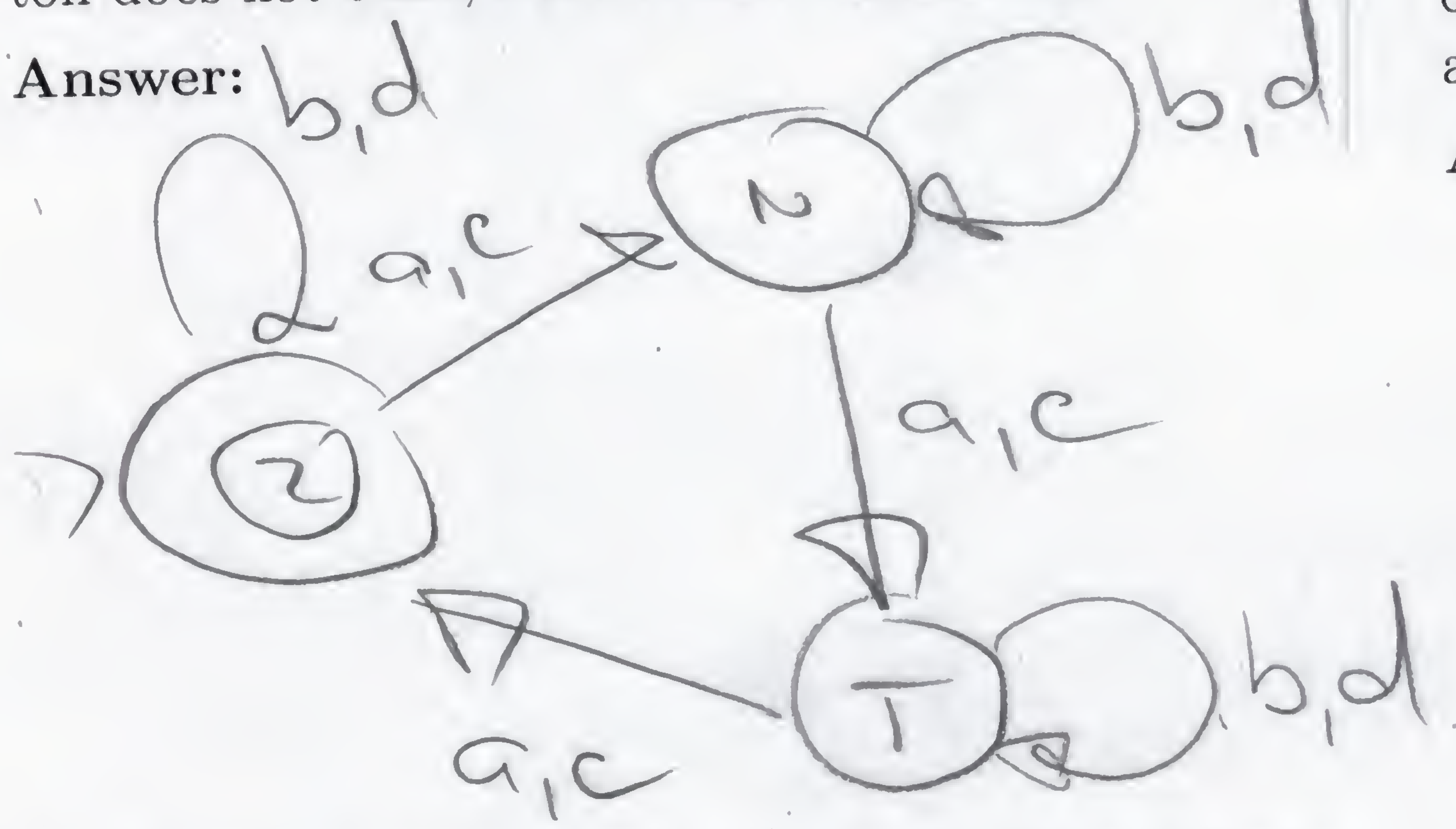
**(a)** Draw a state-transition graph of a finite automaton that accepts the language $L_1$. If such an automaton does not exist, state it and explain why.

**Answer:**



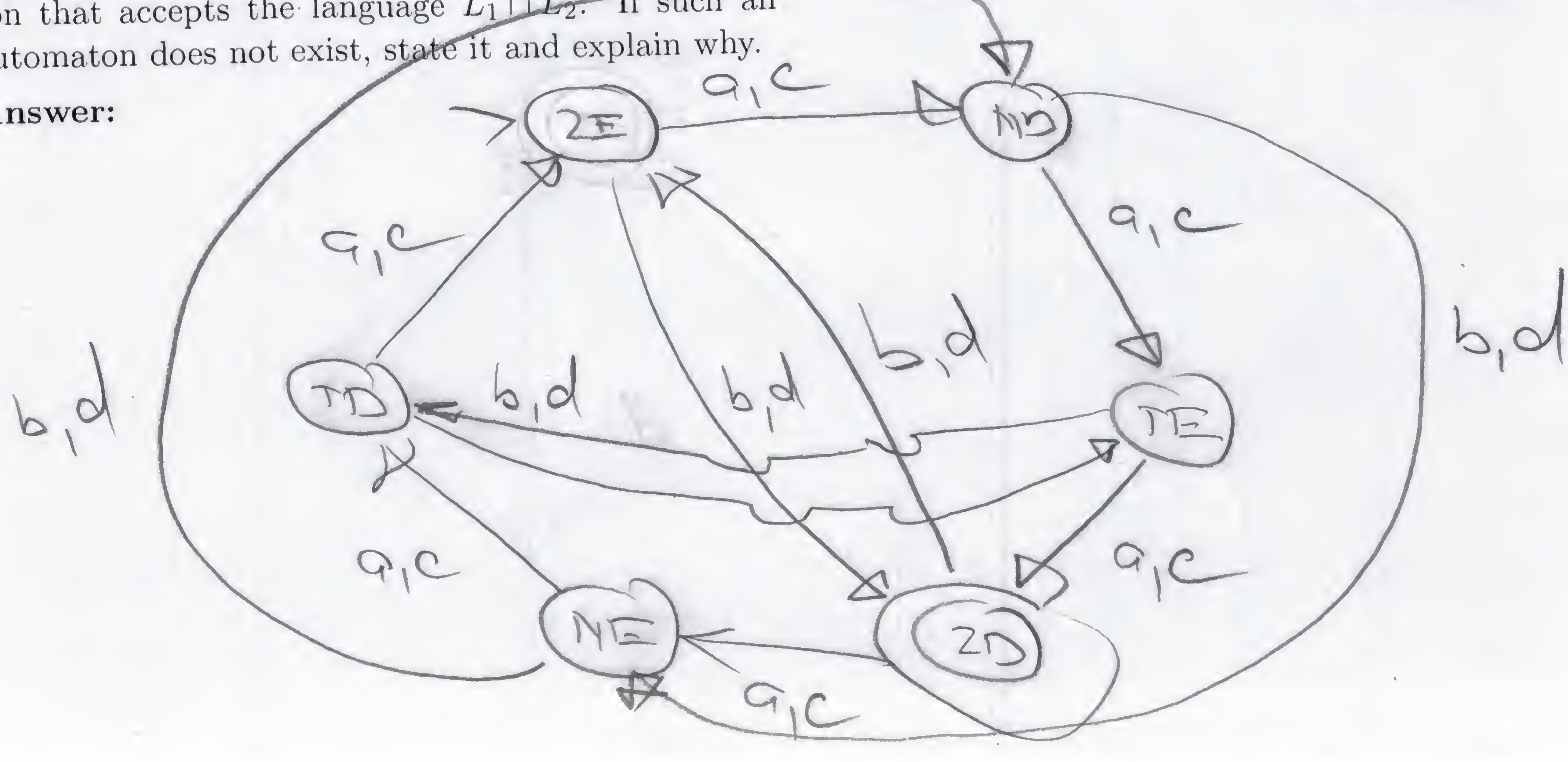

**(b)** Draw a state-transition graph of a finite automaton that accepts the language $L_2$. If such an automaton does not exist, state it and explain why.

**Answer:**




**(c)** Draw a state-transition graph of a finite automaton that accepts the language $L_1 \cap L_2$. If such an automaton does not exist, state it and explain why.

**Answer:**

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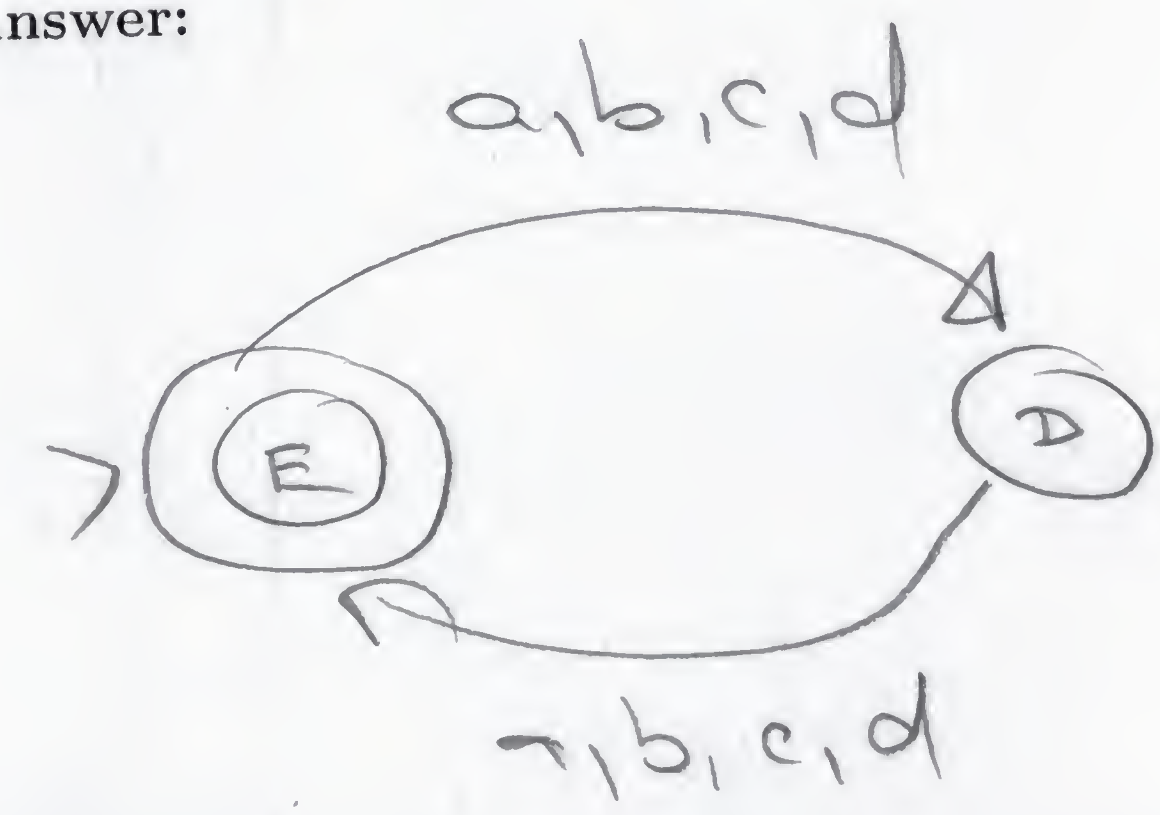
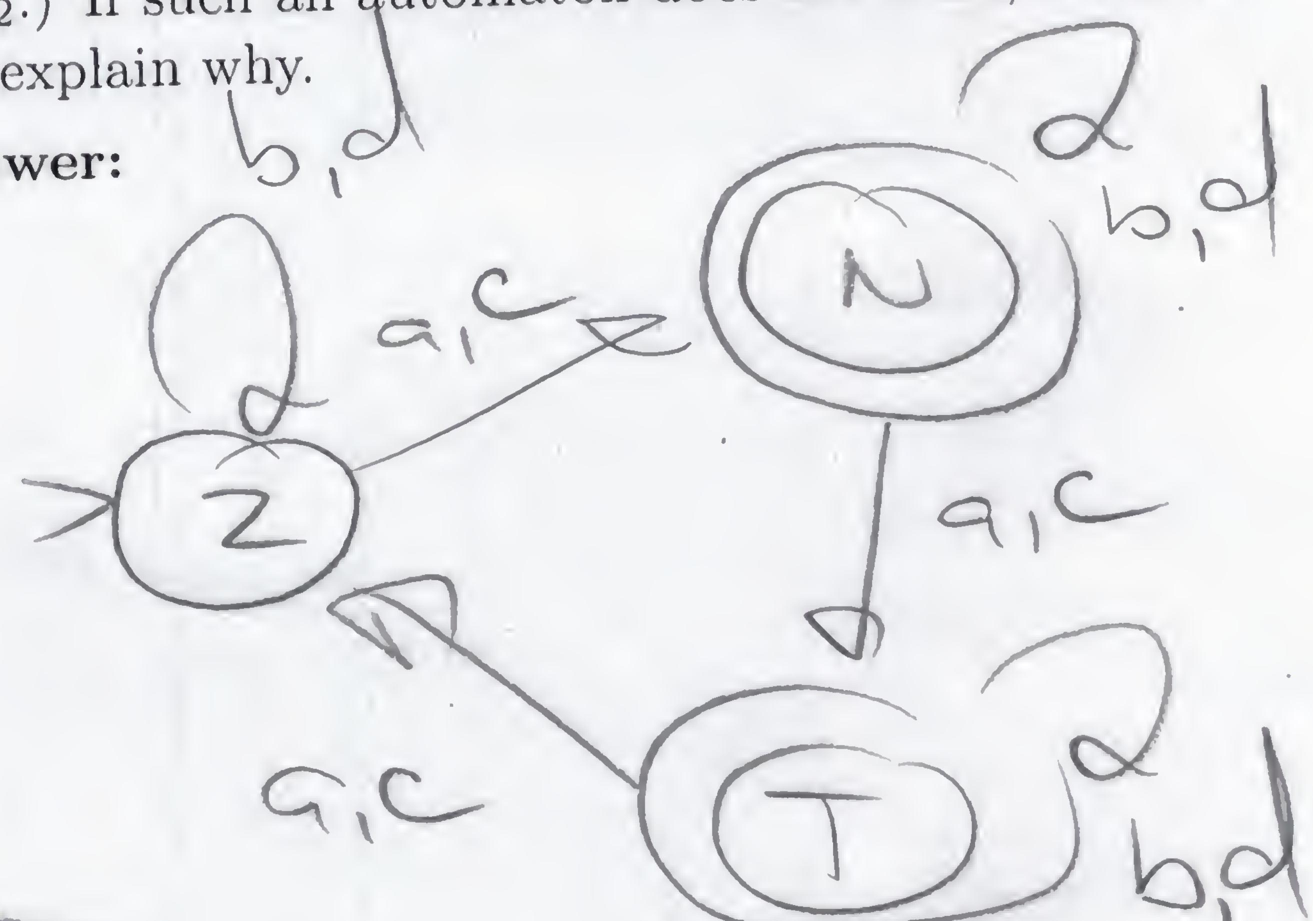
**(d)** Draw a state-transition graph of a finite automaton that accepts the language $\overline{L_1}$ (the complement of $L_1$.) If such an automaton does not exist, state it and explain why.

**Answer:**




**(e)** Draw a state-transition graph of a finite automaton that accepts the language $\overline{L_2}$ (the complement of $L_2$.) If such an automaton does not exist, state it and explain why.

**Answer:**



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**Problem 5** Let $L$ be the set of exactly those strings over the alphabet $\{a, b, c\}$ which satisfy all of the following properties:

1. begins and ends with the same letter;
2. contains exactly two $c$'s.

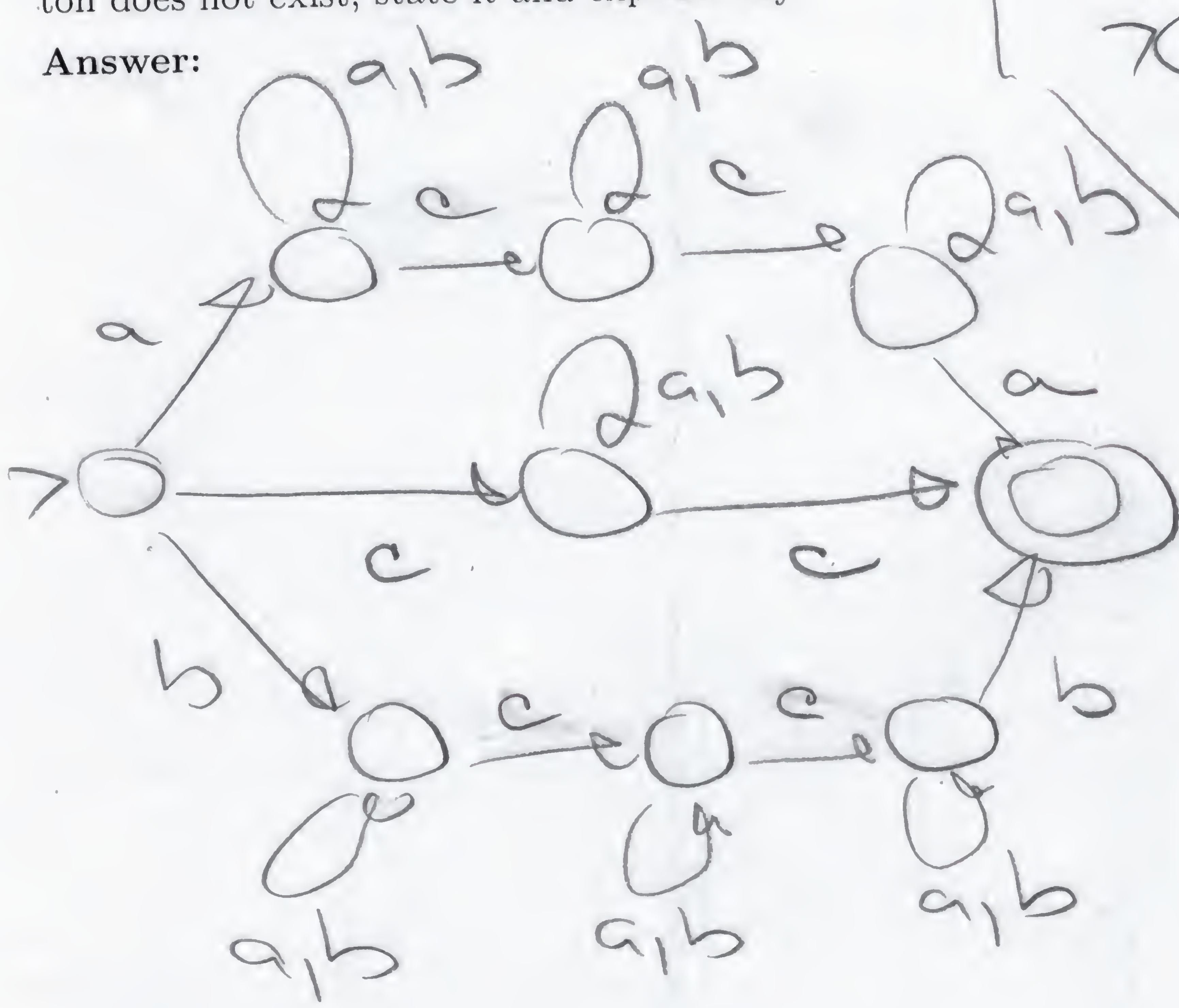
**(a)** Write a regular expression that represents the language $L$. If such a regular expression does not exist, state it and explain why.

**Answer:**

$$a(a \cup b)^* c (a \cup b)^* c (a \cup b)^* a$$
$$\cup$$
$$b(a \cup b)^* c (a \cup b)^* c (a \cup b)^* b$$
$$\cup$$
$$c(a \cup b)^* c$$

**(b)** Draw a state-transition graph of a finite automaton that accepts the language $L$. If such an automaton does not exist, state it and explain why.

**Answer:**




**(c)** Write a complete formal definition of a context-free grammar that generates the language $L$. If such a grammar does not exist, state it and explain why.

**Answer:**

$G = (V, \Sigma, P, S)$

$\Sigma = \{a, b, c\}$

$V = \{S, A, B, K, D\}$

$P$: $S \to A \mid B \mid K$

$A \to aDcDcDa$

$B \to bDcDcDb$

$K \to cDc$

$D \to \lambda \mid DD \mid a \mid b$

**(d)** Draw a state-transition graph of a finite automaton that accepts the language $L \cap \mathbf{c}^*$. If such an automaton does not exist, state it and explain why.

**Answer:**

LAST NAME: ____________________

FIRST NAME: ____________________

**Problem 5** Let $L$ be the set of exactly those strings over the alphabet $\{a, b, c\}$ which satisfy all of the following properties:

1. first letter is either $a$ or $b$;
2. last letter is either $b$ or $c$;
3. first letter is different from the last letter;
4. contains exactly two $c$'s.

**(a)** Write a regular expression that represents the language $L$. If such a regular expression does not exist, state it and explain why.

**Answer:**

$a(a\cup b)^*c(a\cup b)^*c(a\cup b)^*b$

$\cup$

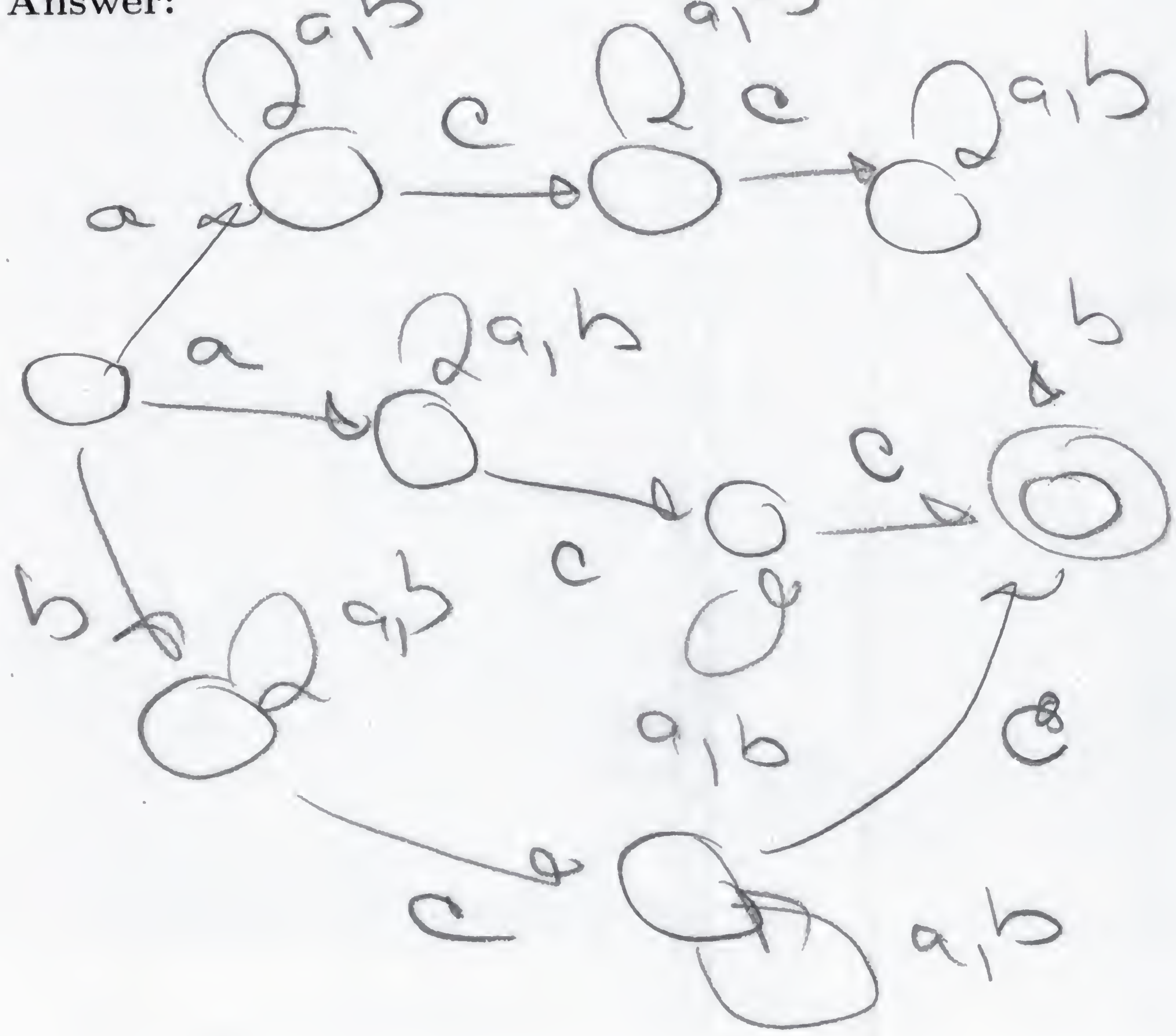
$a(a\cup b)^*c(a\cup b)^*c$

$\cup$

$b(a\cup b)^*c(a\cup b)^*c$

**(b)** Draw a state-transition graph of a finite automaton that accepts the language $L$. If such an automaton does not exist, state it and explain why.

**Answer:**




**(c)** Write a complete formal definition of a context-free grammar that generates the language $L$. If such a grammar does not exist, state it and explain why.

**Answer:**

$G = (V, \Sigma, P, S)$

$\Sigma = \{a, b, c\}$

$V = \{S, A, B, D, E\}$

$S \to A \mid B \mid D$

$A \to aEcEcEb$

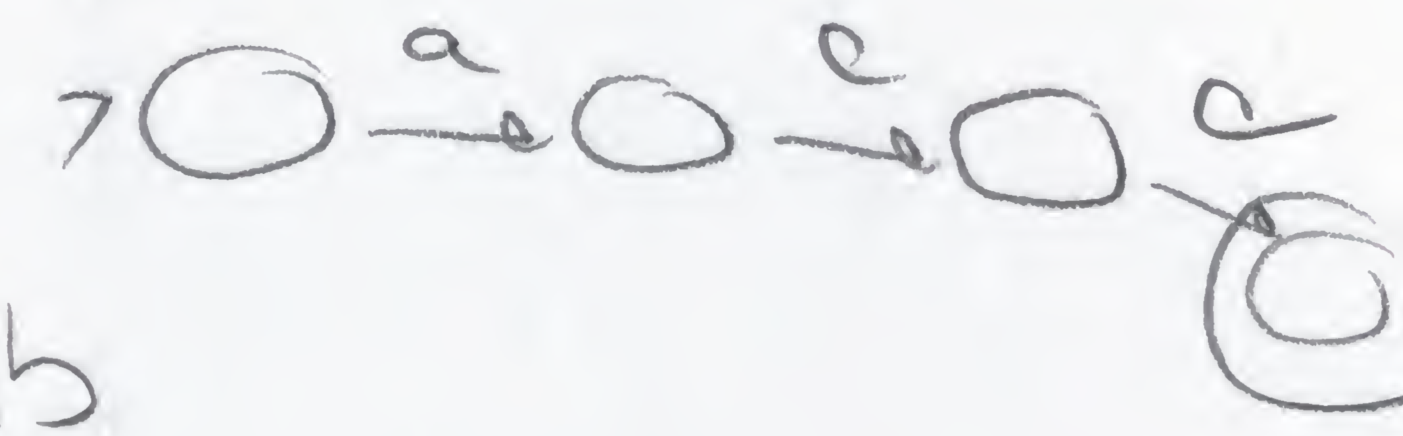
$B \to aEcEc$

$D \to bEcEb$

$E \to \Lambda \mid EE \mid a \mid b$

**(d)** Draw a state-transition graph of a finite automaton that accepts the language $L \cap \boldsymbol{a}\,\boldsymbol{c}^*$. If such an automaton does not exist, state it and explain why.

**Answer:**



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**Problem 6** Let $L_1, L_2$ be languages over the alphabet $\{a, b, c, d, g, e\}$, defined as follows:

$$L_1 = \{g^{3k}\, e^{2i+3}\, d^{2\ell}\, c^{2t+1}\, b^{\ell}\, a^{k}\}$$

$$L_2 = \{c^{2m+3}\, a^{3m+1}\, d^{2n}\, g^{j+2}\, e^{3p}\, b^{j+1}\}$$

where $m, j, n, p, i, k, \ell, t \geq 0$.

**(a)** Write a complete formal definition of a context-free grammar that generates $L_1$. If such a grammar does not exist, state it and explain why.

**Answer:**

$G = (V, S, P, T_1)$

$\Sigma = \{a, b, c, d, g\}$

$V = \{T_1, A, B, D\}$

$P:\ T_1 \to ggg\,T_1 a \mid AB$

$A \to eeA \mid eee$

$B \to ddBb \mid D$

$D \to ccD \mid c$

**(b)** Write a complete formal definition of a context-free grammar that generates $L_2$. If such a grammar does not exist, state it and explain why.

**Answer:**

$G = (V, \Sigma, P, T_2)$

$V = \{T_2, E, F, H, J\}$

$\Sigma = \{a, b, c, d, g\}$

$P:\ T_2 \to EFH$

$E \to ccEaaa \mid ccca$

$F \to ddF \mid \lambda$

$H \to gHb \mid ggJb$

$J \to eeeJ \mid \lambda$

**(c)** Write a complete formal definition of a context-free grammar that generates $(L_1 \cup L_2)^*$. If such a grammar does not exist, state it and explain why.

**Answer:**

$G = (V, \Sigma, P, S)$

$V = \{S, T_1, A, B, D, T_2, E, F, H, J\}$

$\Sigma = \{a, b, c, d\}$

$P:\ S \to \lambda \mid SS \mid T_1 \mid T_2$

$T_1 \to ggg\,T_1 a \mid AB$

$A \to eeA \mid eee$

$B \to ddBb \mid D$

$D \to ccD \mid c$

$T_2 \to EFH$

$E \to ccEaaa \mid ccca$

$F \to ddF \mid \lambda$

$H \to gHb$

$H \to ggJb$

$J \to eeeJ \mid \lambda$

**(d)** Write a complete formal definition of a context-free grammar that generates $L_1^* \cup L_2^*$. If such a grammar does not exist, state it and explain why.

**Answer:**

$G = (V, \Sigma, P, S)$

$\Sigma = \{a, b, c, d, g\}$

$V = \{S, S_1, S_2, T_1, T_2, A, B, D, E, F, H, J\}$

$P:\ S \to S_1 \mid S_2$

$S_1 \to \lambda \mid S_1 S_1 \mid T_1$

$S_2 \to \lambda \mid S_2 S_2 \mid T_2$

$T_1 \to ggg\,T_1 a \mid AB$

$A \to eeA \mid eee$

$B \to ddBb \mid D$

$D \to ccD \mid c$

$T_2 \to EFH$

$E \to ccEaaa \mid ccca$

$F \to ddF \mid \lambda$

$H \to gHb$

$H \to ggJb$

$J \to eeeJ$

$J \to \lambda$

LAST NAME: ____________________

FIRST NAME: ____________________

**Problem 6** Let $L_1, L_2$ be languages over the alphabet $\{a, b, c, d, g, e\}$, defined as follows:

$$L_1 = \{a^{3m+2}\, c^{2m+1}\, e^{2n}\, b^{j+3}\, g^{3p}\, d^{j+2}\}$$

$$L_2 = \{b^k\, g^{2i+1}\, a^{\ell}\, e^{2t+3}\, d^{2\ell}\, c^{3k}\}$$

where $m, j, n, p, i, k, \ell, t \geq 0$.

**(a)** Write a complete formal definition of a context-free grammar that generates $L_1$. If such a grammar does not exist, state it and explain why.

**Answer:**

$G = (V, \Sigma, P, S)$

$\Sigma = \{a, b, c, d, g\}$

$V = \{S, A, B, D, E\}$

P: $S \to ABD$

$A \to aaaAcc \mid aac$

$B \to eeB \mid \lambda$

$D \to bDd \mid bbbEdd$

$E \to gggE \mid \lambda$

**(b)** Write a complete formal definition of a context-free grammar that generates $L_2$. If such a grammar does not exist, state it and explain why.

**Answer:**

$G = (V, \Sigma, P, T)$

$V = \{T, F, H, J\}$

$\Sigma = \{a, b, c, d, g\}$

P: $T \to bTccc \mid FH$

$F \to ggF \mid g$

$H \to aHdd \mid J$

$J \to eeJ \mid eee$

**(c)** Write a complete formal definition of a context-free grammar that generates $L_1^* \cup L_2^*$. If such a grammar does not exist, state it and explain why.

**Answer:**

$G = (V, \Sigma, P, Q)$

$V = \{Q, Q_1, Q_2, S, A, B, D, E, T, F, H, J\}$

$\Sigma = \{a, b, c, d, g\}$

P: $Q \to Q_1 \mid Q_2$

$Q_1 \to \lambda \mid Q_1Q_1 \mid S$

$S \to ABD$

$A \to aaaAcc \mid aac$

$B \to eeB \mid \lambda$

$D \to bDd \mid bbbEdd$

$E \to gggE \mid \lambda$

$T \to bTccc \mid FH$

**(d)** Write a complete formal definition of a context-free grammar that generates $(L_1 \cup L_2)^*$. If such a grammar does not exist, state it and explain why.

**Answer:** $F \to ggF \mid g$

$H \to aHdd \mid J$

$J \to eeJ \mid eee$

$G = \{V, \Sigma, P, Q\}$

$\Sigma = \{a, b, c, d, g\}$

$V = \{Q, S, A, B, D, E, T, F, H, J$

P: $Q \to \lambda \mid QQ \mid S \mid T$

$S \to ABD$

$A \to aaaAcc \mid aac$

$B \to eeB \mid \lambda$

$D \to bDd \mid bbbEdd$

$E \to gggE \mid \lambda$

$T \to bTccc \mid FH$

$F \to ggF$ [illegible]

$H \to aHd$ [illegible]

$H \to J$

$J \to eee$ [illegible]

$J \to eee$ [illegible]

LAST NAME: ______________________

FIRST NAME: ______________________

**Problem 7** Let $L$ be the set of exactly those strings over the alphabet $\{a, b, c, d\}$ which satisfy all of the following properties.

1. the string is a concatenation of four non-empty palindromes;
2. three of the four palindromes have an odd length;
3. one of the four palindromes has an even length;
4. the four palindromes may appear in any order;
5. the middle symbol of each of the three odd-length palindromes is different from $d$;
6. the middle two symbols of the even-length palindrome are different from $a$.

Write a complete formal definition of a context-free grammar that generates the language $L$. If such a grammar does not exist, state it and explain why.

**Answer:**

$G = (V, \Sigma, P, S)$

$\Sigma = \{a, b, c, d\}$

$V = \{S, E, D\}$

$P$: $S \to EDDD \mid DEDD \mid DDED \mid DDDE$

$E \to aEa \mid bEb \mid cEc \mid dEd \mid bb \mid cc \mid dd$

$D \to aDa \mid bDb \mid cDc \mid a \mid b \mid c$

LAST NAME: ______________________

FIRST NAME: ______________________

**Problem 7** Let $L$ be the set of exactly those strings over the alphabet $\{a, b, c\}$ which satisfy all of the following properties.

1. the string is a concatenation of four non-empty palindromes;
2. three of the four palindromes have an even length;
3. one of the four palindromes has an odd length;
4. the four palindromes may appear in any order;
5. the middle symbol of the odd-length palindrome is different from $a$;
6. the middle two symbols of each of the three even-length palindromes are different from $d$.

Write a complete formal definition of a context-free grammar that generates the language $L$. If such a grammar does not exist, state it and explain why.

**Answer:**

$G = (V, \Sigma, P, S)$

$\Sigma = \{a, b, c\}$

$V = \{S, E, D\}$

$P: S \to DEEE \mid EDEE \mid EEDE \mid EEED$

$E \to aEa \mid bEb \mid cEc \mid aa \mid bb \mid cc$

$D \to aDa \mid bDb \mid cDc \mid b \mid c$